Introduction

- This work proposes an effective solution to radio frequency interference (RFI) identification using dynamic power spectrum statistics.
- From the principle of generalized likelihood ratio test, we developed a practical test statistic, $T(x; f_k)$, shown to be $F$-distributed, for narrow-band RFI identification in the presence of Gaussian noise and/or dynamic solar flare.
- The solution is tested on our frequency agile solar radio-telescope (FASR) subsystem test-bed (FST) to demonstrate its feasibility, robustness, and scalability.

System Overview

FASR is a radio telescope array designed for high-resolution imaging spectroscopy with spatial, spectral and temporal characteristics well-matched to solar physics. An artist’s conception of the FASR array of 2m, 6m, and dipole antennas, in a spiral configuration, would appear at the proposed site in CA, USA.

Data Model & GLRT

- Mixed-spectrum solar data consisting:
  - The Background Noise (White Gaussian)
  - The Solar Emission (continuum with gradual PSD)
  - The RFI Component (narrowband spectral lines)
- We formulate the RFI identification (filtered data within a small testing frequency bin) as a hypothesis testing problem.
  - $H_0$: $x[n] = \xi[n]$ for $n = 0, 1, \ldots, N - 1$
  - $H_1$: $x[n] = \cos(2\pi f_k \Delta t) + \xi[n]$ for $n = 0, 1, \ldots, N - 1$
- For the GLRT, we have the detection criterion,
  $$L_G(x) = \frac{p(x; \hat{\alpha}, \hat{\gamma}, f_k, H_1)}{p(x; \hat{\alpha}, \hat{\gamma}, H_0)}$$

The ML estimates of the parameters are given as,
$$\hat{\alpha} = \frac{1}{N} \sum_{n=0}^{N-1} x[n]^2$$
$$\hat{\gamma} = \frac{1}{N} \sum_{n=0}^{N-1} |x[n] - \cos(2\pi f_k \Delta t)|^2$$

Finally, we have,
$$L_G(x) = \left(1 + \frac{\hat{\alpha}}{\hat{\gamma}}\right)^{-N/2} \frac{\hat{\gamma}}{\gamma_0}$$

- Assume that there are $M$ adjacent data blocks available for analysis, to test if RFI is present at certain frequency bin $f_k$,
  - we propose to use,
  $$\hat{\alpha}_k = \frac{1}{M} \sum_{m=0}^{M-1} p(x; f_k, \hat{\alpha}_m)$$
  $$\hat{\gamma}_k = \frac{1}{M} \sum_{m=0}^{M-1} |x[n] - \cos(2\pi f_k \Delta t)|^2$$

Such a procedure is performed through all the frequency bins.

Our Test Statistic for RFI Identification

$$T(x; f_k) = \frac{2p^{(av)}(x; f_k)}{p^{(av)}(x; f_k-1) + p^{(av)}(x; f_k+1)}$$

For background noise
The averaged PSD samples follow a $\chi^2_M$ distribution. Accordingly, $T(x; f_k)$ follows a $F$-distribution with,
$$E(T(x; f_k)) = \frac{2M}{2M - 1} \approx 1 \ (\text{for large } M)$$
$$Var(T(x; f_k)) = \frac{3 \beta M^3 - M}{2(M^2 - 4M^2 + 2M)} \approx \frac{3}{2M} \ (\text{for large } M).$$

For $H_1$ Hypothesis
The averaged PSD sample follow a $\chi^2_M(\lambda_0)$ distribution, with $\lambda_0 = M \beta$. Thus $T(x; f_k)$ is noncentral $F$-distributed with,
$$E(T(x; f_k)) \approx 1 + \frac{2\beta}{M} \ (\text{for large } M).$$
$$Var(T(x; f_k)) \approx \frac{1}{M} \left(\frac{2 + 2\beta + \frac{1}{4M}}{2M^2 + \frac{3}{2M^3}}\right) \ (\text{for large } M).$$

Testing of the proposed test statistic vs. signal duty cycle (varying from 0 to 1) and SNR (varying from 1dB to 10dB) of the simulated RFI.

With a signal duty cycle $\beta$ introduced, we have (for large M),
$$E(T(x; f_k)) \approx 1 + \frac{1 + \frac{1}{4M}}{2M^2 + \frac{3}{2M^3}}$$
$$Var(T(x; f_k)) \approx \frac{3}{2M^2} + \frac{1}{M} \left(\frac{2 + 2\beta + \frac{1}{4M}}{2M^2 + \frac{3}{2M^3}}\right).$$